6. Equilibrium-Core Design

- As discussed in Section 5.3 above, the equilibrium core contains fuel with a distribution of burnups, and therefore also of irradiations, ranging from zero to discharge values.
- Ideally, while the nominal ("average-picture") equilibrium core could be analyzed by averaging, over time, many simulations of a long period of reactor operation, this is not very practical.

6. Equilibrium-Core Design

- Instead, there are two models, representing different levels of approximation,
- which have been used for the analysis of the nominal equilibrium core:
  - the axially homogeneous model and
  - the time-average model.

6. Equilibrium-Core Design

- The axially homogeneous model is an older, cruder model which was developed in the early days of CANDU design and which was used up to about 20 years ago.
- It has been essentially abandoned in favour of the more refined time-average model.
- Although the axially homogeneous model is no longer in routine use, it is instructive to study it, as it introduces useful quantities and concepts.

6.1 Radial Flattening of the Power Distribution

- The flux distribution in the reactor depends on the reactor size and geometry and on the distribution of irradiation.
- Fuel with a high irradiation has low reactivity, and depresses flux in its vicinity.
- Similarly, the neutron flux tends (everything else being equal) to be high in regions where the fuel has low irradiation.
- This fact can be used to “shape” the flux (and power) distributions in the equilibrium core.

6.1 Radial Flattening of the Power Distribution

- **Radial flux (and power) flattening can be achieved by differential fuelling, i.e. taking the fuel to a higher burnup in inner core regions than in outer core regions.**
- **This can be done by adjustment of the relative refuelling rates in the different core regions.**

6.1 Radial Flattening of the Power Distribution

- **In this way, the flux and power in the outer region can be increased, resulting in a greater number of channels having power close to the maximum value.**
- **Thus, a higher total reactor power can be obtained (for a given number of fuel channels) without exceeding the limit on individual channel power.**
- **This reduces the capital cost of the reactor per installed kW.**

6.1 Radial Flattening of the Power Distribution

- **The radial flattening is quantitatively measured by the radial form factor:**

$$\text{Radial form factor} = \frac{\text{Average channel power}}{\text{Maximum channel power}}$$

6.1 Radial Flattening of the Power Distribution

- **Radial flattening is further assisted by the use of adjuster rods (described in Section 2.3),**
- **whose main purpose is in xenon override.**
- **Adjuster rods also provide axial power flattening.**

6.1 Radial Flattening of the Power Distribution

- **Note that while flattening of the power distribution reduce the reactor capital cost,**
- **by reducing the number of channels required to produce a given total power,**
- **it does tend to increase the neutron leakage, which is proportional to the flux gradient at the edge of the core.**
- **This loss of neutrons does have a consequent increase in fuelling cost.**

6.2 Axially Homogeneous Model

- The axially homogeneous model is based on:
- the continuous-refuelling approximation, i.e., the approximation that fuel is pushed continuously along every channel at a constant rate (which may vary from one burnup region to another),
- the bi-directional feature of refuelling in CANDU, i.e., the fact that neighbouring channels are refuelled in opposite directions, and
- the (symmetry) approximation that the flux, as a function of distance from the refuelling end, is equal in neighbouring channels in the same burnup region.

6.2 Axially Homogeneous Model

- With these assumptions, it can be shown that the irradiation, averaged over two neighbouring channels, is independent of axial position in the core. The mathematical treatment is simple and is shown in the next few equations, (6.1)-(6.10) .

6.2 Axially Homogeneous Model

- Consider two neighbouring channels; these are refuelled in opposite directions.
- Suppose that fuel moves continuously at speed r in these two channels (in opposite directions). See Figure 6.1.

6.2 Axially Homogeneous Model

- The fuel irradiation is defined as the product of Westcott flux in the fuel and time.
- Since the fuel moves through the channels and is exposed to different flux values as it moves, the fuel irradiation in each channel at position $x = x_0$ (measured from one end of the reactor, taken as the origin for the axial co-ordinate) must be calculated as the integral over time of fuel flux as fuel travels through the core to $x = x_0$:

6.2 Axially Homogeneous Model

- $\omega_1(x_0) = \int_0^{t_1(x_0)} \hat{\phi}(x(t))dt$ **(6.1)**
- $\omega_2(x_0) = \int_0^{t_2(x_0)} \hat{\phi}(x(t))dt$ **(6.2)**
- **where $t_1(x_0)$ and $t_2(x_0)$ are the times to reach position x_0:**
- $t_1(x_0) = \frac{x_0}{r}$ **(6.3)**
- $t_2(x_0) = \frac{L - x_0}{r}$ **(6.4)**

6.2 Axially Homogeneous Model

- **Use Equations (6.3), (6.4) to rewrite 1 and 2 as integrals over x:**

- $$\omega_1(x_0) = \int_0^{x_0} \hat{\phi}(x)\frac{dx}{r} = \frac{1}{r}\int_0^{x_0} \hat{\phi}(x)\,dx \qquad \textbf{(6.5)}$$

- $$\omega_2(x_0) = \int_L^{x_0} \hat{\phi}(x)(-\frac{dx}{r}) = \frac{1}{r}\int_{x_0}^{L} \hat{\phi}(x)dx \qquad \textbf{(6.6)}$$

6.2 Axially Homogeneous Model

- The average fuel irradiation at $x = x_0$ over the two channels is thus:

$$\omega_{avge}(x_o) = \frac{1}{2}\left[\omega_1(x_0) + \omega_2(x_0)\right]$$

$$= \frac{1}{2r}\left[\int_0^{x_0} \hat{\phi}(x)\,dx + \int_{x_0}^{L} \hat{\phi}(x)\,dx\right]$$

$$= \frac{1}{2r}\int_o^L \hat{\phi}(x)\,dx \qquad (6.7)$$

[The last step requires the symmetry assumption, that the flux shape (as a function of distance from the inlet end of a channel) is the same in the two neighbouring channels .]

6.2 Axially Homogeneous Model

- The right-hand side of Eq. (6.7) does not depend on the position x_0 in the channels.
- This shows that we can treat the average irradiation as uniform along the two channels, equal to one half of the fuel exit irradiation (ω_{exit}) at the channel outlet:

$$\omega_{exit} = \omega_1(L) = \omega_2(0) = \frac{1}{r}\int_o^L \hat{\phi}(x)\,dx \qquad (6.8)$$

6.2 Axially Homogeneous Model

- Therefore it is appropriate within the assumptions for this "axially-homogeneous" model to use uniform basic-lattice properties along the length of the two channels.
- We could define the average cross sections as the values at half the exit irradiation, but a more "correct" treatment defines the average cross section of the fuel as the value which preserves the total reaction rate as the fuel travels through core.

6.2 Axially Homogeneous Model

- Let us label the cross sections in this homogeneous model $\Sigma_i(\text{hom})$, with the subscript i representing the various processes (absorption, production, etc.)
- The cross section which preserves the reaction rate is

$$\Sigma_i(\text{hom}) = \frac{\textit{reaction rate averaged overtime}}{\textit{flux averaged over time}} = \frac{\frac{1}{T}\int_o^T \Sigma_i(\omega(t))\,\hat{\phi}(x(t))\,dt}{\frac{1}{T}\int_o^T \hat{\phi}(x(t))\,dt} \qquad (6.9)$$

where T = transit time of fuel through the core:

$$T = \frac{L}{r}$$

6.2 Axially Homogeneous Model

- **Integrals over time can be replaced by integrals over irradiation ($d\omega = \hat{\phi}\, dt$), and we get:**

$$\Sigma_i\,(\text{hom}) = \frac{1}{\omega_{\text{exit}}} \int_0^{\omega_{\text{exit}}} \Sigma_i\,(\omega)\, \mathrm{d}\omega \qquad (6.10)$$

where ω_{exit} is specific to the channel pair.

6.2 Axially Homogeneous Model

- Most often, the axially homogeneous model is used with only two large core regions, the inner core and the outer core, each with a different value of exit. Figure 6.2 shows a two-region model for the CANDU 6.
- The cross sections defined in Eq. (6.10) are calculated, corresponding to the value of exit in each core region, directly in POWDERPUFS-V, since Eq. (6.10) is identical to the “reaction-rate-averaged” equation.

6.2 Axially Homogeneous Model

- As can be seen, the calculational procedure for the axially homogeneous model is simple.
- It consists of a simple integral, which can be computed within POWDERPUFS-V.
- The resulting values of lattice cross sections are applied uniformly (both radially and axially) in large regions with a given value of exit irradiation ω_{exit}.

6.2 Axially Homogeneous Model

- The application is simple, and was adequate in the early days.
- However, the continuous-refuelling approximation does not take into account the axial non-uniformity induced by real refuelling schemes. For this reason, the axially homogeneous model has been largely abandoned in favour of the time-average model, described in the next section.

6.3 Time-Average Model

- In the time-average model, the lattice cross sections are averaged over the individual residence (dwell) time of the fuel at each point (fuel-bundle position) in the core.
- This allows the effect of the actual refuelling scheme used (e.g. 8-bundle shift, 4-bundle shift, etc.) to be captured.
- Calculations are performed in the *TIME-AVER module of RFSP. The mathematical framework of this module will be described next, for the specific case of an 8-bundle-shift refuelling scheme as an example.

6.3 Time-Average Model

- Time-average nuclear cross sections are defined at each bundle position in core by averaging over the irradiation range [ω_{in}, ω_{out}] "experienced" over time by fuel at that position.
- ω_{in} is the value of fuel irradiation when the fuel enters that position in core,
- and ω_{out} is the fuel irradiation when the fuel leaves that position.

6.3 Time-Average Model

- As in the axially homogeneous model, the correct definition of cross section is the one which preserves the average reaction rate.
- In this case, because we are looking at an individual fuel-bundle location,
- the irradiation range is not 0 to exit,
- but instead ω_{in} to ω_{out}. These are values specific to that location.

6.3 Time-Average Model

- For example, the time-average thermal neutron absorption cross section at some core position r, $\Sigma_{a2}^{t.a.}(r)$, is

$$\Sigma_{a2}^{t.a.}(r) = \frac{1}{(\omega_{out} - \omega_{in})} \int_{\omega_{in}}^{\omega_{out}} \Sigma_{a2}(\omega)\, d\omega \qquad (6.11)$$

- The basic lattice cross sections inside the integral sign are determined as functions of instantaneous irradiation
- using POWDERPUFS-V, incorporated within RFSP as the *POWDERPUF module.

6.3 Time-Average Model

- Now, in the time-average model, let $\phi_{\varphi K}$ be the time-average fuel flux
- at axial (bundle) position k
- in channel j.
- k ranges from 1 to 12 since there are 12 bundles per channel, and
- j ranges over the channels, e.g. from 1 to 380 in the CANDU 6.
- The bundle position is labelled jk for short

6.3 Time-Average Model

- Let also T_j be the average time interval between refuellings of channel j (also known as the dwell time of channel j).
- Then the irradiation increment which the fuel at position jk will experience over its residence time at that position will be (flux*time)

$$\Delta\omega_{jk} = \phi_{jk}.T_j \qquad (6.12)$$

6.3 Time-Average Model

- If the fuel entered position jk with an irradiation $\omega_{in,jk}$,
- then its exit irradiation from that position, $\omega_{out,jk}$, is given by

$$\omega_{out,jk} = \omega_{in,jk} + \Delta\omega_{jk}$$

$$= \omega_{in,jk} + \phi_{jk}.T_j \qquad (6.13)$$

6.3 Time-Average Model

- When a channel is refuelled with an 8-bundle shift, the first 8 positions in the channel receive fresh fuel
- and the entrance irradiations for positions 9-12
- are simply the exit irradiations from positions 1-4 respectively.
- Thus we can write in this case (see Figure 6.3):

$$\omega_{in,jk} = 0 \qquad k = 1,...,8 \qquad (6.14a)$$

$$\omega_{in,jk} = \omega_{out,j(k-8)} \qquad k = 9,...,12 \qquad (6.14b)$$

6.3 Time-Average Model

- More generally, for an N-bundle-shift refuelling scheme, these equations would become

$$\omega_{in,jk} = 0 \qquad k = 1,\ldots,N \qquad (6.15a)$$

$$\omega_{in,jk} = \omega_{out,j(k-N)} \qquad k = (N+1),\ldots,12 \qquad (6.15b)$$

6.3 Time-Average Model

- In addition to the refuelling scheme, we have other degrees of freedom in the time-average model.
- These degrees of freedom are inherent to CANDU.
- They are the values of exit irradiation $\omega_{exit,j}$ for the various channels j.
- In principle there are as many degrees of freedom as there are channels.
- (Of course the values of exit irradiation are not totally free, but are collectively constrained by the requirement to obtain a critical reactor.)

6.3 Time-Average Model

- The **relative** values of $\omega_{exit,j}$ can be used to "shape" the flux to a desired reference distribution.
- The exit irradiations are related to the flux in the following way,
- written here explicitly for the 8-bundle-shift case.
- In the eight-bundle-shift refuelling scheme, bundles 5 to 12 leave the core at each refuelling,
- so by definition of exit irradiation

$$\omega_{exit,j} = \frac{1}{8}\sum_{k=5}^{12} \omega_{out,jk} \qquad (6.16)$$

6.3 Time-Average Model

- **Using Equation (6.13) which relates the outgoing irradiations to the incoming irradiations, we can develop Eq. (6.16):**

$$\omega_{exit,j} = \frac{1}{8}\sum_{k=5}^{12}\left(\omega_{in,jk} + \phi_{jk}\,T_j\right)$$

and then subdivide the sum over positions 5-8, 9-12:

$$\omega_{exit,j} = \frac{1}{8}\left[\sum_{k=5}^{8}\left(\omega_{in,jk} + \phi_{jk}\,T_j\right) + \sum_{k=9}^{12}\left(\omega_{in,jk} + \phi_{jk}\,T_j\right)\right] \qquad \textbf{(6.17)}$$

6.3 Time-Average Model

- We continue the development;
- Since positions 1-8 receive fresh fuel, the incoming irradiations $\omega_{in,jk}$ are zero for k= 1 to 8, and drop out of the first sum
- Also, positions 9-12 receive fuel from positions 1-4, so the corresponding $\omega_{in,jk}$ can be replaced by $\omega_{out,j(k-8)}$, and these by flux*time
- i.e., by applying Eq. (6.15), we can rewrite Eq. (6.17) as

$$\omega_{exit,j} = \frac{1}{8}\left[\sum_{k=5}^{8}\phi_{jk}T_j + \sum_{k=1}^{4}\phi_{jk}T_j + \sum_{k=9}^{12}\phi_{jk}T_j\right] = \frac{T_j}{8}\sum_{k=1}^{12}\phi_{jk} \qquad (6.18)$$

6.3 Time-Average Model

- It is easy to derive the generalization of this result to an N-bundle-shift refuelling scheme:

$$\omega_{exit,j} = \frac{T_j}{N} \sum_{k=1}^{12} \phi_{jk} \qquad (6.19)$$

- We can invert this equation to solve for the dwell time T_j:

$$T_j = \frac{N\,\omega_{exit,j}}{\sum_{k=1}^{12} \phi_{jk}} \qquad (6.20)$$

6.3 Time-Average Model

- We now have all the equations required for the time-average flux distribution to be calculated. They are:
- the finite-difference form of the time-independent neutron diffusion equation to solve for the flux distribution,
- Eq. (6.20) for the dwell time for each channel,
- Eqs. (6.13) and (6.14) for $\omega_{in,jk}$ and $\omega_{out,jk}$ for each bundle in core,
- Eq. (6.11) (and similar equations for the other cross sections) to calculate the time-average lattice properties.

6.3 Time-Average Model

- This set of equations must be solved,
- using as input the user-specified target exit irradiations $\omega_{exit,j}$.
- In order to shape the flux to desired values,
- and also to take account of the presence of extra "hardware" (device locators, etc.), mostly at the bottom of the calandria) which introduces localized absorption,
- typical time-average RFSP models now use many (rather than only 2) irradiation regions; see Figure 6.4.

6.3 Time-Average Model

- Consistency must be achieved between:
  - the flux,
  - the channel dwell times,
  - the individual-bundle irradiation ranges $[\omega_{in}, \omega_{out}]$, and
  - the lattice properties,
- an iterative scheme between the solution of the diffusion equation and the other equations is employed until all quantities converge.
- Figure 6.5 illustrates the iterative scheme of calculations.

6.3 Time-Average Model

- Typically, several iterations around the loop are required for convergence.
- The user must verify that the solution gives:
  - a critical reactor, and
  - the desired flux shape, measured say by the degree of flattening, or radial form factor.
- If these results are not attained, the values of exit irradiation $\omega_{exit,j}$ for the various regions must be adjusted to obtain the desired result.

6.3 Time-Average Model

- The time-average model is useful at the design stage, to determine:
- the reference three-dimensional power distribution,
- the expected refuelling frequency of each channel (or its inverse, the channel dwell time), and
- the expected value of discharge burnup for the various channels.

6.3 Time-Average Model

- The time-average model is also useful beyond the design stage.
- Instead of doing a large number of calculations for core "snapshots", the time-average model can be used to analyze the response of the core to various perturbations, such as:
  - device movements,
  - hypothetical loss-of-coolant accidents
  - xenon transients
  - etc.
- In such analyses, the irradiations are not iterated upon, since the situation is a perturbation.

6.3 Time-Average Model

- Figure 6.6 shows the time-average channel-power distribution obtained with
- a two-region model for the CANDU 6, where the exit irradiations (approximately 1.8 and 1.6 n/kb in the outer and inner cores respectively)
- were chosen to produce a radial form factor (ratio of average to maximum channel power) of 0.83.

6.3 Time-Average Model

- This radial form factor gives a maximum time-average channel power of about 6.52 MW,
- a value substantially lower than the maximum licensed channel power, 7.3 MW.
- However, remember that there must be sufficient margin above the maximum time-average power to accommodate the refuelling ripple,
- which could be of the order of 10%.
- The average in-core fuel irradiation in this model is approximately 1 n/kb.

6.3 Time-Average Model

- Figure 6.7 shows the channel dwell times for the same CANDU-6 time-average calculation.
- The dwell times in the inner core range typically between 150 and 160 full-power days (FPD).
- In the outer core, the dwell times present a large variation, from about 135 FPD for channels just outside the inner core (where the flux is still high but the exit irradiation is, by design, lower than in the inner core)
- to almost 300 FPD for some channels at the outermost periphery of the core.

6.3 Time-Average Model

- The time-average calculation also provides other results, such as:
- the rate of core reactivity decay in the absence of refuelling:
  - this is ~0.4 milli-k/FPD for the CANDU 6 with 37-element fuel, and
- the average channel-visit rates per core region
  - (for a given channel, the visit rate is the inverse of the dwell time)

6.3 Time-Average Model

- It is instructive to look at a typical neutron balance in the CANDU-6 equilibrium core. This is displayed in Figure 6.8.
- More than 45% of fission neutrons originate from fissions in plutonium.
- Thus, plutonium contributes approximately half the fission energy produced in a CANDU reactor.
- Actually, in fuel near the exit burnup, plutonium contributes about three-quarters of the fission energy generated.

6.3 Time-Average Model

- Fast fissions account for 56 fission neutrons out of 1,000.
- Total neutron leakage is 29 neutrons lost per 1000 neutrons born in fission, representing a 29-milli-k loss (6 milli-k from fast leakage, 23 milli-k from thermal leakage).
- Resonance absorption in ^{238}U represents a loss of almost 90 milli-k.
- Parasitic absorption in non-fuel components of the lattice represents a 63-milli-k loss.

6.3 Time-Average Model

- The time-average model is an excellent representation of the equilibrium core, on the average.
- However, the one thing that the time-average model does not include (by design) is the refuelling ripple:
- In the time-average model no channel is “fresh”, and no channel is “old”, thus the refuelling ripple is absent.
- The next chapter deals with the effects of channel refuellings.